

GOLD-PLATED MODE OF CP-VIOLATION IN DECAYS OF  $B_c$  MESON FROM QCD SUM RULES

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The model-independent method based on the triangle ideology is implemented to extract the CKM-matrix angle  $\gamma$  in the decays of doubly heavy long-lived meson  $B_c$ . We analyze a color structure of diagrams and conditions to reconstruct two reference-triangles by tagging the flavor and CP eigenstates of  $D^0 \leftrightarrow \bar{D}^0$  mesons in the fixed exclusive channels. The characteristic branching ratios are evaluated in the framework of QCD sum rules.

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## I. INTRODUCTION

The  $B_c$  meson first observed by the CDF collaboration at FNAL [1] is expected to be copiously produced in the future experiments at hadron colliders [2] with facilities oriented to the study of fine effects in the heavy quark interactions such as the parameters of CP-violation and charged weak current mixing<sup>1</sup>. So, one could investigate the spectroscopy, production mechanism and decay features of  $B_c$  [4, 5, 6] with the incoming sample of several billion events. In such circumstances, in addition to the current success in the experimental study of decays with the CP-violation in the gold-plated mode of neutral  $B$ -meson by the BaBar and Belle collaborations [7] allowing one to extract the CKM-matrix angle  $\beta$  in the unitarity triangle, a possible challenge is whether one could get an opportunity to extract some information about the CKM unitarity triangle from the  $B_c$  physics in a model independent way or not. The theoretical principal answer is one can do it. Indeed, there is an intriguing opportunity to extract the angle  $\gamma$  in the model-independent way using the strategy of reference triangles [8] in the decays of doubly heavy hadrons. This ideology for the study of CP-violation in  $B_c$  decays was originally offered by M.Masetti [9], independently investigated by R.Fleischer and D.Wyler [10] and extended to the case of doubly heavy baryons<sup>2</sup> in [12].

Let us point out necessary conditions to extract the CP-violation effects in the model-independent way.

1. Interference. The measured quantities have to involve the amplitudes including both the CP-odd and CP-even phases.
2. Exclusive channels. The hadronic final state has to be fixed in order to isolate a definite flavor contents and, hence, the definite matrix elements of CKM matrix, which can exclude the interference of two CP-odd phases with indefinite CP-even phases due to strong interactions at both levels of the quark structure and the interactions in the final state.
3. Oscillations. The definite involvement of the CP-even phase is ensured by the oscillations taking place in the systems of neutral  $B$  or  $D$  mesons, wherein the CP-breaking effects can be systematically implemented.
4. Tagging. Once the oscillations are involved, the tagging of both the flavor and CP eigenstates is necessary for the complete procedure.

The gold-plated modes in the decays of neutral  $B$  mesons involve the oscillations of mesons themselves and, hence, they require the time-dependent measurements. In contrast, the decays of doubly heavy hadrons such as the  $B_c$  meson and  $\Xi_{bc}$  baryons with the neutral  $D^0$  or  $\bar{D}^0$  meson in the final state do not require the time-dependent measurements. The triangle ideology is based on the direct determination of absolute values for the set of four decays, at least: the decays of hadron in the tagged  $D^0$  meson, the tagged  $\bar{D}^0$  meson, the tagged CP-even state<sup>3</sup> of  $D^0$ , and the decay of

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<sup>1</sup> See, for instance, the program on the B physics at Tevatron [3].

<sup>2</sup> A review on the physics of doubly heavy baryons is given in [11].

<sup>3</sup> The CP-odd states of  $D^0$  can be used, too. However, their registration requires the detection of CP-even state of  $K^0$ , which can be complicated because of a detector construction, say, by a long base of  $K^0$  decay beyond a tracking system.

the anti-hadron into the tagged CP-even state of  $D^0$ . To illustrate, let us consider the decays of

$$B_c^+ \rightarrow D^0 D_s^+, \quad \text{and} \quad B_c^+ \rightarrow \bar{D}^0 D_s^+.$$

The corresponding diagrams with the decay of  $\bar{b}$ -quark are shown in Figs. 1 and 2. We stress that two diagrams of the decay to  $D^0$  have the additional negative sign caused by the Pauli interference of two charmed quarks, which, however, completely compensated after the Fierz transformation for the corresponding Dirac matrices.

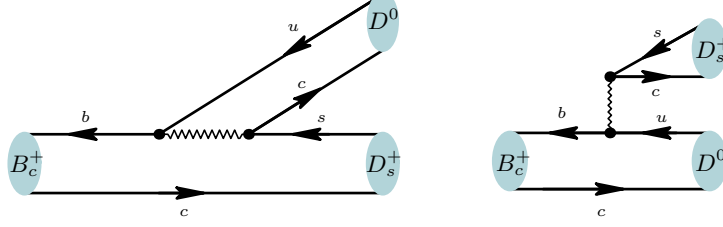


FIG. 1: The diagrams of  $\bar{b}$ -quark decay contributing to the weak transition  $B_c^+ \rightarrow D^0 D_s^+$ .

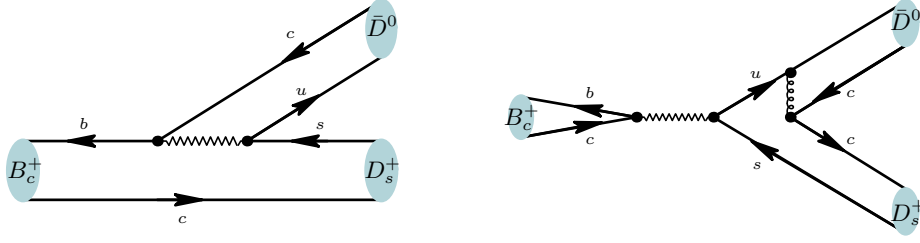


FIG. 2: The diagrams of  $\bar{b}$ -quark decay contributing to the weak transition  $B_c^+ \rightarrow \bar{D}^0 D_s^+$ .

The exclusive modes make the penguin terms to be excluded, since the penguins add an even number of charmed quarks, i.e. two or zero, while the final state contains two charmed quarks including one from the  $\bar{b}$  decay and one from the initial state. However, the diagram with the weak annihilation of two constituents, i.e. the charmed quark and beauty anti-quark in the  $B_c^+$  meson, can contribute in the next order in  $\alpha_s$  as shown in Fig. 2 for the given final state. Nevertheless, we see that such the diagrams have the same weak-interaction structure as at the tree level. Therefore, they do not break the consideration under interest. The magnitude of  $\alpha_s$ -correction to the absolute values of corresponding decay widths is discussed in Section II.

Thus, the CP-odd phases of decays under consideration are determined by the tree-level diagrams shown in Figs. 1 and 2. Therefore, we can write down the amplitudes in the following form:

$$\mathcal{A}(B_c^+ \rightarrow D^0 D_s^+) \stackrel{\text{def}}{=} \mathcal{A}_D = V_{ub}^* V_{cs} \cdot \mathcal{M}_D, \quad \mathcal{A}(B_c^+ \rightarrow \bar{D}^0 D_s^+) \stackrel{\text{def}}{=} \mathcal{A}_{\bar{D}} = V_{cb}^* V_{us} \cdot \mathcal{M}_{\bar{D}}, \quad (1)$$

where  $\mathcal{M}_{\bar{D}, D}$  denote the CP-even factors depending on the dynamics of strong interactions. Using the definition of angle  $\gamma$

$$\gamma \stackrel{\text{def}}{=} -\arg \left[ \frac{V_{ub} V_{cs}^*}{V_{cb} V_{us}^*} \right],$$

for the CP-conjugated channels<sup>4</sup> we find

$$\mathcal{A}(B_c^- \rightarrow \bar{D}^0 D_s^-) = e^{-2i\gamma} \mathcal{A}_D, \quad \mathcal{A}(B_c^- \rightarrow D^0 D_s^-) = \mathcal{A}_{\bar{D}}. \quad (2)$$

<sup>4</sup> For the sake of simplicity we put the overall phase of  $\arg V_{cb} V_{us}^* = 0$ , which corresponds to fixing the representation of the CKM matrix, e.g. by the Wolfenstein form [13].

We see that the corresponding widths for the decays to the flavor tagged modes coincide with the CP-conjugated ones. However, the story can be continued by using the definition of CP-eigenstates for the oscillating  $D^0 \leftrightarrow \bar{D}^0$  system<sup>5</sup>,

$$D_{1,2} = \frac{1}{\sqrt{2}}(D^0 \pm \bar{D}^0),$$

so that we straightforwardly get

$$\sqrt{2}\mathcal{A}(B_c^+ \rightarrow D_s^+ D_1) \stackrel{\text{def}}{=} \sqrt{2}\mathcal{A}_{D_1} = \mathcal{A}_D + \mathcal{A}_{\bar{D}}, \quad (3)$$

$$\sqrt{2}\mathcal{A}(B_c^- \rightarrow D_s^- D_1) \stackrel{\text{def}}{=} \sqrt{2}\mathcal{A}_{D_1}^{\text{CP}} = e^{-2i\gamma}\mathcal{A}_D + \mathcal{A}_{\bar{D}}. \quad (4)$$

The complex numbers entering (3) and (4) establish two triangles with the definite angle  $2\gamma$  between the vertex positions as shown in Fig. 3. Thus, due to the unitarity, the measurement of four absolute values

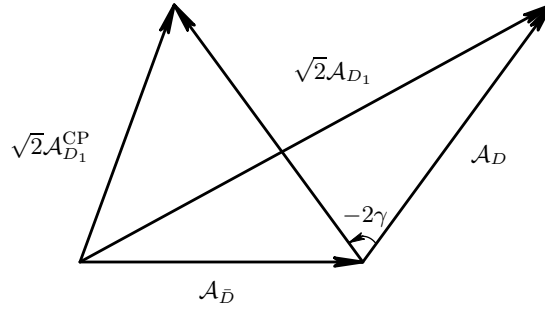


FIG. 3: The reference-triangles.

$$\begin{aligned} |\mathcal{A}_D| &= |\mathcal{A}(B_c^+ \rightarrow D_s^+ D^0)|, & |\mathcal{A}_{\bar{D}}| &= |\mathcal{A}(B_c^+ \rightarrow D_s^+ \bar{D}^0)|, \\ |\mathcal{A}_{D_1}| &= |\mathcal{A}(B_c^+ \rightarrow D_s^+ D_1)|, & |\mathcal{A}_{D_1}^{\text{CP}}| &= |\mathcal{A}(B_c^- \rightarrow D_s^- D_1)|, \end{aligned} \quad (5)$$

can constructively reproduce the angle  $\gamma$  in the model-independent way.

The above triangle-ideology can be implemented for the analogous decays to the excited states of charmed mesons in the final state.

The residual theoretical challenge is to evaluate the characteristic widths or branching fractions. We address this problem and analyze the color structure of amplitudes. So, we find that the matrix elements under interest have the different magnitudes of color suppression, so that at the tree level we get  $\mathcal{A}_D \sim O(\sqrt{N_c})$  and  $\mathcal{A}_{\bar{D}} \sim O(1/\sqrt{N_c})$ , while the ratio of relevant CKM-matrix elements,

$$\left| \frac{V_{ub}V_{cs}^*}{V_{cb}V_{us}^*} \right| \sim O(1)$$

with respect to the small parameter of Cabibbo angle,  $\lambda = \sin\theta_C$ , which one can easily find in the Wolfenstein parametrization

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}.$$

<sup>5</sup> The suppressed effects of CP-violation in the oscillations of neutral  $D$  mesons are irrelevant here, and we can neglect them in the sound way.

Nevertheless, the interference of two diagrams in the decays of  $B_c^+$  to the  $D^0$  meson is destructive, and the absolute values of the amplitudes  $\mathcal{A}_D$  and  $\mathcal{A}_{\bar{D}}$  become close to each other. Thus, we expect that the sides of the reference-triangles are of the same order of magnitude, which makes the method to be an attractive way to extract the angle  $\gamma$ .

In Section II we classify the diagrams for the decays of doubly heavy meson  $B_c^+$  by the color and weak-interaction structures. Section III is devoted to the numerical estimates in the framework of QCD sum rules. The results are summarized in Conclusion.

## II. COLOR STRUCTURES

In the framework of  $1/N_c$ -expansion we have got the following scaling rules of color structures in the processes with the hadrons composed of the quark and anti-quark:

1. The meson wavefunction

$$\Psi_M \sim \frac{1}{\sqrt{N_c}} \delta^i_j.$$

2. The coupling constant

$$\alpha_s \sim \frac{1}{N_c}.$$

3. The Casimir operators

$$C_A = N_c, \quad C_F = \frac{N_c^2 - 1}{2N_c} \sim O(N_c).$$

4. The Fierz relation for the generators of  $SU(N_c)$  group in the fundamental representation

$$t^A{}^i{}_j t^A{}^k{}_m = \frac{C_F}{N_c} \delta^i{}_m \delta^k{}_j - \frac{1}{N_c} t^A{}^i{}_m t^A{}^k{}_j.$$

Next, the non-leptonic weak Lagrangian has a form typically given by the following term [14]:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{2\sqrt{2}} V_{cb}(\bar{b}^i \Gamma_\mu c_j) V_{us}^*(\bar{u}^k \Gamma^\mu s_l) C_\pm (\delta^i{}_j \delta^k{}_l \pm \delta^i{}_l \delta^k{}_j) + \dots \quad (6)$$

where  $\Gamma_\mu = \gamma_\mu(1 - \gamma_5)$ , and the Wilson coefficients

$$C_\pm \sim O(1)$$

in the  $1/N_c$ -expansion.

Then, we can proceed with the analysis of decays under interest.

Two diagrams shown in Fig. 1 scale with the different order in  $1/N_c$ , so that

$$\mathcal{A}_{1D} \sim \frac{1}{\sqrt{N_c}}, \quad \mathcal{A}_{2D} \sim \sqrt{N_c}.$$

More definitely we get the color factors

$$\mathcal{F}_{1D}^c = \sqrt{N_c} a_2, \quad \mathcal{F}_{2D}^c = \sqrt{N_c} a_1, \quad (7)$$

where

$$a_1 = \frac{1}{2N_c} [C_+(N_c + 1) + C_-(N_c - 1)], \quad (8)$$

$$a_2 = \frac{1}{2N_c} [C_+(N_c + 1) - C_-(N_c - 1)]. \quad (9)$$

Nevertheless, we have to calculate both diagrams in order to take into account their interference which is not suppressed by any kinematical factors except the color factor of  $1/N_c$ .

Two diagrams shown in Fig. 2 also scale with the different order in  $1/N_c$ , so that

$$\mathcal{A}_{1\bar{D}} \sim \frac{1}{\sqrt{N_c}}, \quad \mathcal{A}_{2\bar{D}} \sim \sqrt{N_c}.$$

More definitely we get the color factors

$$\mathcal{F}_{1\bar{D}}^c = \sqrt{N_c} a_2, \quad \mathcal{F}_{2\bar{D}}^c = \sqrt{N_c} a_1 C_F 4\pi\alpha_s, \quad (10)$$

where we have explicitly included the  $\alpha_s$  correction. However, we can easily find that due to the virtualities of quarks and gluons the kinematical suppression of second diagram in Fig. 2 is given by the factor of

$$\mathcal{S} \sim |\tilde{\Psi}(0)|^2 \frac{\alpha_s(k_g^2)}{k_g^2} \frac{N_c}{\Delta E_q},$$

where

$$|\tilde{\Psi}(0)|^2 \sim \Lambda_{\text{QCD}}^3$$

is the characteristic value of wavefunction, the gluon virtuality is determined by the expression

$$k_g^2 = m_c^2(v_2 + v_D)^2 = m_c^2 \frac{M_1^2 - (M_2 - M_3)^2}{M_2 M_3} \approx (m_b + m_c)^2 + O(m_{s,d}/m_{c,b}) \gg \Lambda_{\text{QCD}}^2,$$

with  $M_{1,2,3}$  being the masses of the mesons in the initial and final states, respectively, and the virtuality of quark line connected to the virtual gluon is of the order of

$$\Delta E_q \sim m_c.$$

Therefore,

$$\mathcal{S} \sim \frac{\Lambda_{\text{QCD}}^3}{m_{c,b}^3} \frac{1}{\ln m_{c,b}/\Lambda_{\text{QCD}}} \ll 1.$$

Therefore, the above  $\alpha_s$ -corrections can be neglected to the leading order in the  $1/m_Q$ -expansion.

Finally, in this section we have analyzed the color and weak-interaction structures of decay amplitudes and isolate those of the largest magnitude, while the numerical estimates are presented in the next section.

### III. NUMERICAL ESTIMATES

In this section we use the machinery of QCD sum rules [15, 16] in order to calculate the widths and branching ratios for the gold-plated modes under study.

The calculations of exclusive non-leptonic modes usually involves the approximation of factorization [17], which, as expected, can be quite accurate for the  $B_c$ , since the quark-gluon sea is suppressed in the heavy quarkonium. Thus, the important parameters are the factors  $a_1$  and  $a_2$  in the non-leptonic weak lagrangian, which depend on the normalization point suitable for the  $B_c$  decays. In this way, we, first, calculate the form factors in the semileptonic transitions [18, 19, 20] and, second, evaluate the non-leptonic matrix elements in the factorization approach.

We accept the following convention on the normalization of wave functions for the hadron states under study, i.e. for the pseudoscalar ( $P$ ) and vector ( $V$ ) states:

$$\langle 0 | \mathcal{J}_\mu | P \rangle = -i f_P p_\mu, \quad (11)$$

$$\langle 0 | \mathcal{J}_\mu | V \rangle = \epsilon_\mu f_V M_V, \quad (12)$$

where  $f_{P,V}$  denote the leptonic constants, so that they are positive,

$$f_{P,V} > 0,$$

$p_\mu$  is a four-momentum of the hadron,  $\epsilon_\mu$  is a polarization vector of  $V$ ,  $M_V$  is its mass, and the current is composed of the valence quark fields constituting the hadron

$$\mathcal{J}_\mu = \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2.$$

In this respect we can easily apply the ordinary Feynman rules for the calculations of diagrams, so that the quark-meson vertices in the decay channel are chosen with the following spin structures:

$$\Gamma_P = \frac{i}{\sqrt{2}} \gamma_5, \quad \Gamma_V = -\frac{1}{\sqrt{2}} \epsilon_\mu.$$

Then, we get general expressions for the hadronic matrix elements of weak currents in the exclusive decays of  $P \rightarrow P'$  and  $P \rightarrow V$  with the definitions of form factors given by the formulae

$$\langle P'(p_2) | \mathcal{J}_\mu | P(p_1) \rangle = f_+ p_\mu + f_- q_\mu, \quad (13)$$

$$\frac{1}{i} \langle V(p_2) | \mathcal{J}_\mu | P(p_1) \rangle = i F_V \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p^\alpha q^\beta + F_0^A \epsilon_\mu^* + F_+^A (\epsilon^* p_1) p_\mu + F_-^A (\epsilon^* p_1) q_\mu, \quad (14)$$

where  $q_\mu = (p_1 - p_2)_\mu$  and  $p_\mu = (p_1 + p_2)_\mu$ . The form factors  $f_\pm$  are dimensionless, while  $F_V$  and  $F_\pm^A$  has a dimension of inverse energy,  $F_0^A$  is of the energy dimension. In the case of nonrelativistic description for both initial and final meson states we expect that

$$f_+ > 0, \quad f_- < 0, \quad F_V > 0, \\ F_0^A > 0, \quad F_+^A < 0, \quad F_-^A > 0.$$

It is important to note that for the pseudoscalar state the hermitian conjugation in (11) does not lead to the change of sign in the right hand side of equation because of the prescription accepted, since the conjugation of imaginary unit takes place with the change of sign for the momentum of meson (the transition from the out-state to in-one). The same speculations show that the spin structure of matrix element in the quark-loop order does not involve a functional dependence of form factors on the transfer momentum squared except of  $F_0^A$ , so that we expect that the simplest modelling in the form of the pole dependence can be essentially broken for  $F_0^A$ , while the other form factors are fitted by the pole model in a reasonable way.

Following the standard procedure for the evaluation of form factors in the framework of QCD sum rules [16], in the  $B_c$  decays we consider the three-point functions

$$\Pi_\mu(p_1, p_2, q^2) = i^2 \int dx dy e^{i(p_2 \cdot x - p_1 \cdot y)} \cdot \langle 0 | T \{ \bar{q}_2(x) \gamma_5 q_1(x) \mathcal{J}_\mu(0) \bar{b}(y) \gamma_5 c(y) \} | 0 \rangle, \quad (15)$$

$$\Pi_{\mu\nu}^\mathcal{J}(p_1, p_2, q^2) = i^2 \int dx dy e^{i(p_2 \cdot x - p_1 \cdot y)} \cdot \langle 0 | T \{ \bar{q}_2(x) \gamma_\mu q_1(x) \mathcal{J}_\nu(0) \bar{b}(y) \gamma_5 c(y) \} | 0 \rangle, \quad (16)$$

where  $\bar{q}_2(x) \gamma_5 q_1(x)$  and  $\bar{q}_2(x) \gamma_\nu q_1(x)$  denote the interpolating currents for the final states mesons.

The standard procedure for the evaluation of correlators and form factors is described in [18, 19, 20, 21]. The most important notes are the following [18, 19, 20, 21]:

- For the heavy quarkonium  $\bar{b}c$ , where the relative velocity of quark movement is small, an essential role is taken by the Coulomb-like  $\alpha_s/v$ -corrections.
- We have found that the normalizations of leptonic constants for the heavy quarkonia are fixed by the appropriate choice of effective constant for the coulomb exchange  $\alpha_s^C$ , while the stability is very sensitive to the prescribed value of heavy quark mass. Thus, these parameters of sum rules are extracted from the two-point QCD sum rules with a quite good accuracy.
- In the framework of two-point sum rules for the heavy-light channel the fixed value of threshold energy  $E_c$  determines the binding energy of heavy quark in the meson,  $\bar{\Lambda} \approx 0.63$  GeV, which yields the same value of mass for the beauty quark,  $m_b \approx 4.6$  GeV, as it was determined from the analysis of two-point sum rules in the  $\bar{b}b$  channel. However, taking into account the second order corrections in  $1/m_c$ , we find that the mass of charmed quark is shifted to the value of  $m_c \approx 1.2$  GeV in the heavy-light channel in comparison with the  $\bar{c}c$  states. Thus, in the transition of  $B_c$  to the charmed meson we put the mass of spectator charmed quark equal to  $m_c \approx 1.2$  GeV.

- In the framework of the effective theory of heavy quarks with the expansion in the inverse masses of the heavy quarks [22, 23, 24, 25], the spin symmetry relations between the form factors in the soft limit of zero recoil momentum can be derived [20].
- We take the following ordinary ratios of leptonic constants for the vector and pseudoscalar states and for the heavy-strange mesons:

$$\frac{f_{B^*}}{f_B} \approx \frac{f_{D^*}}{f_D} \approx 1.11, \quad \frac{f_{B_s}}{f_B} \approx \frac{f_{D_s}}{f_D} \approx 1.16,$$

which agree with both the lattice computations [26] and the estimates in the framework of potential models taking into account relativistic corrections [27].

- The leptonic constant of  $B_c$  is taken from a scaling relation for the heavy quarkonia [28].

TABLE I: The form factors of various transitions calculated in the framework of QCD sum rules at  $q^2 = 0$  in comparison with the estimates in the potential model (PM) of [29].

Transition	$f_+$ , [PM]	$f_-$ , [PM]	$F_V$ , [PM] (GeV <sup>-1</sup> )	$F_0^A$ , [PM] (GeV)	$F_+^A$ , [PM] (GeV <sup>-1</sup> )	$F_-^A$ , [PM] (GeV <sup>-1</sup> )
$B_c \rightarrow D^{(*)}$	0.32, [0.29]	-0.34, [-0.37]	0.20, [0.21]	3.6, [3.6]	-0.062, [-0.060]	0.10, [0.16]
$B_c \rightarrow D_s^{(*)}$	0.45, [0.43]	-0.43, [-0.56]	0.24, [0.27]	4.7, [4.7]	-0.077, [-0.071]	0.13, [0.20]

Our estimates are summarized in Table I extracted from [21], where for the sake of comparison we expose the results obtained in the potential model [29], which parameters are listed in Appendix B of ref.[20]. In the potential model the most reliable results are expected at zero recoil of meson in the final state of transition, since the wave functions are rather accurately calculable at small virtualities of quarks composing the meson. We take the predictions of the potential model at zero recoil and evolve the values of form factors to zero transfer squared in the model with the pole dependence

$$F_i(q^2) = \frac{F_i(0)}{1 - q^2/M_{i,\text{pole}}^2},$$

making use of numerical values of  $M_{i,\text{pole}}$  shown in Table II. We stress the fact that the potential model points to the approximately constant value of the form factor  $F_0^A$  because of additional kinematical dependence in the transition of  $B_c \rightarrow D^*$  and  $B_c \rightarrow D_s^*$ .

TABLE II: The pole masses used in the model for the form factors in various transitions.

Transition	$M_{\text{pole}}[f_+]$ , GeV	$M_{\text{pole}}[f_-]$ , GeV	$M_{\text{pole}}[F_V]$ , GeV	$M_{\text{pole}}[F_0^A]$ , GeV	$M_{\text{pole}}[F_+^A]$ , GeV	$M_{\text{pole}}[F_-^A]$ , GeV
$B_c \rightarrow D^{(*)}$	5.0	5.0	6.2	$\infty$	6.2	6.2
$B_c \rightarrow D_s^{(*)}$	5.0	5.0	6.2	$\infty$	6.2	6.2

Next, we investigate the validity of spin-symmetry relations in the  $B_c$  decays to  $D^{(*)}$  and  $D_s^{(*)}$ . The results of estimates for the  $f_{\pm}$  evaluated by the symmetry relations with the inputs given by the form factors  $F_V$  and  $F_0^A$  extracted from the sum rules are presented in Table III in comparison with the values calculated in the framework of sum rules.

TABLE III: The comparison of sum rule results for the form factors  $f_{\pm}$  with the values obtained by the spin symmetry with the inputs of  $F_0^A$  and  $F_V$  extracted from the QCD sum rules. The values of pole masses are also explicitly shown.

Transition	Form factor	Sum Rules	Spin symmetry	$M_{\text{pole}}[f_{\pm}]$ , GeV	$M_{\text{pole}}[F_V]$ , GeV
$B_c \rightarrow D^{(*)}$	$f_+$	0.32	0.31	4.8	6.2
$B_c \rightarrow D^{(*)}$	$f_-$	-0.34	-0.36	4.8	6.2
$B_c \rightarrow D_s^{(*)}$	$f_+$	0.45	0.45	4.8	6.2
$B_c \rightarrow D_s^{(*)}$	$f_-$	-0.43	-0.51	4.8	6.2

We have found that the uncertainty in the estimates is basically determined by the variation of pole masses in the  $q^2$ -dependencies of form factors, which govern the evolution from the zero recoil point to the zero transfer squared.

TABLE IV: Exclusive non-leptonic decay widths of the  $B_c$  meson,  $\Gamma$  in  $10^{-15}$  GeV. The  $\bar{b}$ -quark decays with  $c$ -quark spectator.

Class	Mode	$\Gamma$ [21]	$\Gamma$ [30]	$\Gamma$ [31]	$\Gamma$ [32]	$\Gamma$ [33]	$\Gamma$ [34]
II	$B_c^+ \rightarrow D^+ \bar{D}^0$	$1.9 a_2^2$	$0.633 a_2^2$	$0.664 a_2^2$	$2.72 a_2^2$	$0.753 a_2^2$	$0.15 a_2^2$
	$B_c^+ \rightarrow D^+ \bar{D}^{*0}$	$2.75 a_2^2$	$0.762 a_2^2$	$0.695 a_2^2$	$2.10 a_2^2$	$1.925 a_2^2$	$0.13 a_2^2$
	$B_c^+ \rightarrow D^{*+} \bar{D}^0$	$1.8 a_2^2$	$0.289 a_2^2$	$0.653 a_2^2$	$0.86 a_2^2$	$0.399 a_2^2$	$1.46 a_2^2$
	$B_c^+ \rightarrow D^{*+} \bar{D}^{*0}$	$12.0 a_2^2$	$0.854 a_2^2$	$1.080 a_2^2$	$1.32 a_2^2$	$1.95 a_2^2$	$2.4 a_2^2$
	$B_c^+ \rightarrow D_s^+ \bar{D}^0$	$0.18 a_2^2$	$0.0415 a_2^2$	$0.0340 a_2^2$		$0.0405 a_2^2$	$0.01 a_2^2$
	$B_c^+ \rightarrow D_s^+ \bar{D}^{*0}$	$0.25 a_2^2$	$0.0495 a_2^2$	$0.0354 a_2^2$		$0.101 a_2^2$	$0.009 a_2^2$
	$B_c^+ \rightarrow D_s^{*+} \bar{D}^0$	$0.17 a_2^2$	$0.0201 a_2^2$	$0.0334 a_2^2$		$0.0222 a_2^2$	$0.087 a_2^2$
	$B_c^+ \rightarrow D_s^{*+} \bar{D}^{*0}$	$0.93 a_2^2$	$0.0597 a_2^2$	$0.0564 a_2^2$		$0.109 a_2^2$	$0.15 a_2^2$

So, the variation of  $M_{\text{pole}}[f_{\pm}]$  in the range of  $4.8 - 5$  GeV for the transitions of  $B_c \rightarrow D^{(*)}$  and  $B_c \rightarrow D_s^{(*)}$  results in the 30%-uncertainty in the form factors presented in Table III.

The QCD SR estimates for the non-leptonic decays of  $\bar{b}$ -quark in  $B_c$  as obtained in [21] and the present work give the widths represented in Tables IV and V in comparison with the values calculated in the potential models. The sum rule predictions are significantly greater than the estimates of the potential models for the transitions with the color permutation, i.e. for the class II processes<sup>6</sup> with the factor of  $a_2$ .

Further, for the transitions, wherein the interference is significantly involved, the class III processes, we find that the absolute values of different terms given by the squares of  $a_1$  and  $a_2$  calculated in the sum rules are greater than the estimates of potential models. We stress that under fixing the definitions of hadron state phases as described in the beginning of section III, we have found that the Pauli interference has determined the negative sign of two amplitudes with  $a_1$  and  $a_2$ , however, the relevant Fierz transformation has led to the complete cancellation of the Pauli interference effect, and the relative sign of two amplitudes in the modes under consideration is positive in agreement with the results of potential models listed in Table V. Taking into account the negative value of  $a_2$  with respect to  $a_1$ , we see that all of decays shown in Table V should be suppressed in comparison with the case of the interference switched off. The characteristic values of effects caused by the interference is presented in Table VI, where we put the widths in the form

$$\Gamma = \Gamma_0 + \Delta\Gamma, \quad \Gamma_0 = x_1 a_1^2 + x_2 a_2^2, \quad \Delta\Gamma = z a_1 a_2.$$

Then, we conclude that the interference can be straightforwardly tested in the listed decays, wherein its significance reaches about 40%.

TABLE V: Exclusive non-leptonic decay widths of the  $B_c$  meson,  $\Gamma$  in  $10^{-15}$  GeV, the symbol  $\star$  marks the result of this work. The  $\bar{b}$ -quark decays involving the Pauli interference with the  $c$ -quark spectator.

Class	Mode	$\Gamma$ [ $\star$ ]	$\Gamma$ [33]
III	$B_c^+ \rightarrow D^+ D^0$	$(0.023 a_1 + 0.023 a_2)^2$	$(0.0147 a_1 + 0.0146 a_2)^2$
	$B_c^+ \rightarrow D^+ D^{*0}$	$(0.022 a_1 + 0.025 a_2)^2$	$(0.0107 a_1 + 0.0234 a_2)^2$
	$B_c^+ \rightarrow D^{*+} D^0$	$(0.025 a_1 + 0.022 a_2)^2$	$(0.0233 a_1 + 0.0106 a_2)^2$
	$B_c^+ \rightarrow D^{*+} D^{*0}$	$(0.051 a_1 + 0.051 a_2)^2$	$(0.0235 a_1 + 0.0235 a_2)^2$
	$B_c^+ \rightarrow D_s^+ D^0$	$(0.11 a_1 + 0.14 a_2)^2$	$(0.0689 a_1 + 0.672 a_2)^2$
	$B_c^+ \rightarrow D_s^+ D^{*0}$	$(0.11 a_1 + 0.15 a_2)^2$	$(0.0503 a_1 + 0.106 a_2)^2$
	$B_c^+ \rightarrow D_s^{*+} D^0$	$(0.12 a_1 + 0.13 a_2)^2$	$(0.101 a_1 + 0.0498 a_2)^2$
	$B_c^+ \rightarrow D_s^{*+} D^{*0}$	$0.067 a_1^2 + 0.706 a_2^2 + 0.14 a_1 a_2$	$(0.104 a_1 + 0.110 a_2)^2$

<sup>6</sup> The class I processes, which amplitudes are proportional to the factor of  $a_1$ , without the color permutations in the effective lagrangian are not involved in the modes under consideration.



TABLE VI: The effect of interference in the exclusive non-leptonic decay widths of the  $B_c$  meson with the  $c$ -quark as spectator at  $a_1^b = 1.14$  and  $a_2^b = -0.20$ .

Mode	$\Delta\Gamma/\Gamma_0, \%$
$B_c^+ \rightarrow D^+ D^0$	-34
$B_c^+ \rightarrow D^+ D^{*0}$	-38
$B_c^+ \rightarrow D^{*+} D^0$	-30
$B_c^+ \rightarrow D^{*+} D^{*0}$	-34
$B_c^+ \rightarrow D_s^+ D^0$	-43
$B_c^+ \rightarrow D_s^+ D^{*0}$	-45
$B_c^+ \rightarrow D_s^{*+} D^0$	-37
$B_c^+ \rightarrow D_s^{*+} D^{*0}$	-55

The predictions of QCD sum rules for the exclusive decays of  $B_c$  are summarized in Table VII at the fixed values of factors  $a_{1,2}$  and lifetime. For the sake of completeness and comparison we show the estimates for the channels with the neutral  $D$  meson and charged one  $D^+$  as well as for the vector states in addition to the pseudoscalar ones.

First, we see that the similar decay modes without the strange quark in the final state can be, in principle, used for the same extraction of CKM angle  $\gamma$ , however, this channels are more problematic from the methodic point of view, since the sides of reference-triangles significantly differ from each other<sup>7</sup>, so that the measurements have to be extremely accurate in order to get valuable information on the angle. Indeed, we should accumulate a huge statistics for the dominant mode in order to draw any conclusion on the consistency of triangle with a small side.

Second, the decay modes with the vector neutral  $D$  meson in the final state are useless for the purpose of the CKM measurement under the approach discussed. However, the modes with the vector charged  $D^*$  and  $D_s^*$  mesons can be important for the procedure of  $\gamma$  extraction. This note could be essential for the mode with  $D^{*+} \rightarrow D^0 \pi^+$  and  $D^0 \rightarrow K^- \pi^+$ , but, in this case, the presence of neutral charmed meson should be carefully treated in order to avoid the misidentification with the primary neutral charmed meson. In other case, we should use the mode with the neutral pion  $D^{*+} \rightarrow D^+ \pi^0$ , which detection in an experimental facility could be problematic. The same note is applicable for the vector  $D_s^{*+}$  meson, which radiative electromagnetic decay is problematic for the detection, too, since the photon could be loosed. However, the lose of the photon for the fully reconstructed  $D_s^+$  and  $B_c^+$  does not disturb the analysis.

TABLE VII: Branching ratios of exclusive  $B_c^+$  decays at the fixed choice of factors:  $a_1^b = 1.14$  and  $a_2^b = -0.20$  in the non-leptonic decays of  $\bar{b}$  quark. The lifetime of  $B_c$  is appropriately normalized by  $\tau[B_c] \approx 0.45$  ps.

Mode	BR, $10^{-6}$	Mode	BR, $10^{-6}$
$B_c^+ \rightarrow D^+ \bar{D}^0$	53	$B_c^+ \rightarrow D^+ D^0$	0.32
$B_c^+ \rightarrow D^+ \bar{D}^{*0}$	75	$B_c^+ \rightarrow D^+ D^{*0}$	0.28
$B_c^+ \rightarrow D^{*+} \bar{D}^0$	49	$B_c^+ \rightarrow D^{*+} D^0$	0.40
$B_c^+ \rightarrow D^{*+} \bar{D}^{*0}$	330	$B_c^+ \rightarrow D^{*+} D^{*0}$	1.59
$B_c^+ \rightarrow D_s^+ \bar{D}^0$	4.8	$B_c^+ \rightarrow D_s^+ D^0$	6.6
$B_c^+ \rightarrow D_s^+ \bar{D}^{*0}$	7.1	$B_c^+ \rightarrow D_s^+ D^{*0}$	6.3
$B_c^+ \rightarrow D_s^{*+} \bar{D}^0$	4.5	$B_c^+ \rightarrow D_s^{*+} D^0$	8.5
$B_c^+ \rightarrow D_s^{*+} \bar{D}^{*0}$	26	$B_c^+ \rightarrow D_s^{*+} D^{*0}$	40.4

In the above estimates we put the following values of parameters:

- the leptonic constants:  $f_D = 0.22$  GeV,  $f_{D^*} = 0.24$  GeV,  $f_{D_s} = 0.24$  GeV,  $f_{D_s^*} = 0.27$  GeV;
- the CKM elements:  $|V_{ub}| = 0.003$ ,  $|V_{cb}| = 0.04$ ,  $|V_{cs}| = 0.975$ ,

<sup>7</sup> The ratio of widths is basically determined by the factor of  $|V_{cb}V_{ud}a_2|^2/|V_{ub}V_{cd}a_1|^2 \sim 110$ , if we ignore the interference effects.

so that the numbers can be appropriately scaled at other values of input parameters.

In the BTeV [3] and LHCb [35] experiments one expects the  $B_c$  production at the level of several billion events. Therefore, we predict  $10^4 - 10^5$  decays of  $B_c$  in the gold-plated modes under interest. The experimental challenge is the efficiency of detection. One usually get a 10%-efficiency for the observation of distinct secondary vertices outstanding from the primary vertex of beam interaction. Next, we have to take into account the branching ratios of  $D_s$  and  $D^0$  mesons. This efficiency crucially depends on whether we can detect the neutral kaons and pions or not. So, for the  $D_s$  meson the corresponding branching ratios grow from 4% (no neutral  $K$  and  $\pi$ ) to 25%. The same interval for the neutral  $D^0$  is from 11 to 31%. The detection of neutral kaon is necessary for the measurement of decay modes into the CP-odd state  $D_2$  of the neutral  $D^0$  meson, however, we can omit this cross-check channel from the analysis dealing with the CP-even state of  $D_1$ . The corresponding intervals of branching ratios reachable by the experiment are from 0.5 to 1.3% for the CP-even state and from 1.5 to 3.8% for the CP-odd state of  $D^0$ . The pessimistic estimate for the product of branching ratios is about  $2 \cdot 10^{-4}$ , which results in 2 – 20 reconstructed events. Thus, an acceptance of experimental facility and an opportunity to detect neutral pions and kaons as well as reliable estimates of total cross section for the  $B_c$  production in hadronic collisions are of importance in order to make expectations more accurate.

#### IV. CONCLUSION

In this work we have shown how the reference-triangle ideology can be used for the model-independent extraction of CKM-matrix angle  $\gamma$  from the set of branching ratios of doubly heavy meson  $B_c$  exclusively decaying to the neutral  $D$  mesons. Tagging the flavor and CP-eigenstates of such the  $D$  mesons allows one to avoid the uncertainties caused by the QCD dynamics of quarks.

We have estimated the characteristic branching ratios in the framework of QCD sum rules, which yields the values of the order of

$$\mathcal{B}[B_c^+ \rightarrow D_s^+ \bar{D}^0] \approx 5 \cdot 10^{-6}.$$

Accepting the above value, and putting the efficiency of tagging procedure equal to 0.5% for the neutral charmed meson and 4% for the charmed strange meson in the final state as well as the vertex reconstruction efficiency equal to 10%, we can expect the observation of about 10 reconstructed events per year at the LHC collider in such the experiment like LHCb or in BTeV experiment at FNAL.

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